Conditionally prepared photon and quantum imaging

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ABSTRACT

We discuss a classical model allowing one to visualize and characterize the optical mode of the single photon generated by means of a conditional measurement on a biphoton produced in parametric down-conversion. The model is based on Klyshko's advanced wave interpretation, but extends beyond it, providing a precise mathematical description of the advanced wave. The optical mode of the conditional photon is shown to be identical to the mode of the classical difference-frequency field generated due to nonlinear interaction of the partially coherent advanced wave with the pump pulse. With this "nonlinear advanced wave model" most coherence properties of the conditional photon become manifest, which permits one to intuitively understand many recent results, in particular, in quantum imaging.

Keywords: Parametric down-conversion, conditional preparation, quantum imaging

1. INTRODUCTION

Preparation of single-photon states by means of conditional measurements on a biphoton generated through parametric down-conversion has been proposed and tested experimentally in 1986 by Hong and Mandel[1] as well as Grangier, Roger and Aspect[2] and has since become the workhorse for many quantum optics experiments. The idea of the method is based on the fact that in parametric down-conversion, the photons are necessarily born in pairs. The two generated photons can be separated into two emission channels according to their propagation direction, wavelength and/or polarization. Detection of a photon in one of the emission channels (labeled *trigger*) indicates that the quantum ensemble in the remaining (*signal*) channel is also a single-photon state.

This method has recently been utilized by the Konstanz quantum optics group, who has prepared the signal photon in a well-defined, highly pure spatiotemporal mode which can be matched with, coupled into, or caused to interfere with, a classical laser mode. Based on this achievement, we have demonstrated homodyne tomography of the single-photon Fock state[3] and applied the developed know-how to implementing new tools of quantum-optical information technology.[4]

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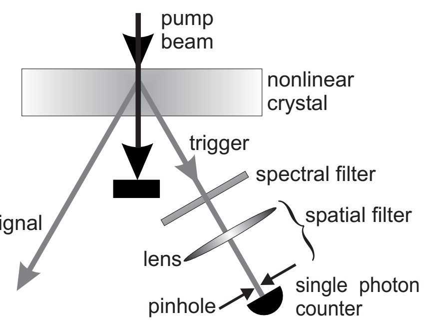

Figure 1. Preparation of single photons by conditional measurements on a biphoton state.

Conditional preparation of a photon from a down-converted pair is a highly complex quantum-mechanical process which involves a collapse of a hyperentangled biphoton state onto a particular signal mode upon the measurement in the trigger channel. When trying to develop insight into this process and relevant experimental techniques,[5] we have found the visualization tool proposed by D. N. Klyshko[6–8] to be of great help. This is the concept of *advanced waves* which assumes that "one of the two detectors, say number 2, at the moment of registration of a photon t_2 emits back in time and space a short δ-like pulse... This pulse interacts within the excited (in a coherent state) atom* and the latter emits a growing wave... with a converted carrier frequency".[6] According to Klyshko, one can sometimes think of the conditionally prepared photon as a wave resulting from the nonlinear interaction between the pump and the fictitious advanced wave.

In the author's opinion, it is unfortunate that Klyshko's ingenious idea has not been developed further in subsequent years. Indeed, many experimental schemes that explore nonclassical correlations in a down-converted pair can be reformulated in a way compatible with the advanced wave model. Within this scope are experiments on entanglement in polarization,[9] frequency-time,[10] angular-momentum[11] and time-of-arrival[12] domains, four-photon interference,[13] Einstein-Podolsky-Rosen-type nonlocality,[14] quantum imaging[15,16] and many others. Our understanding of entanglement could be greatly enhanced if the visual power of the Klyshko model were employed to its full extent.

Furthermore, as demonstrated in our earlier theoretical work, the optical mode of the conditionally prepared photon is in fact *completely identical* to that of the difference-frequency field generated by a properly defined advanced wave.[5] In other words, the advanced wave concept is not merely an informal visual tool, but a rigorous mathematical model which possesses analytic capability that approaches that of the canonical quantum theory. In the present paper, we review the theory associated with the advanced wave concept and discuss its applicability in various experimental situations.

2. THE CONDITIONALLY PREPARED SINGLE PHOTON

We start with a general calculation of the quantum state of a single photon state prepared by a conditional measurement on a PDC biphoton state (Fig. 1). We restrict our consideration to the pulsed regime. In all calculations in this paper, we neglect polarization entanglement (polarization is assumed to be well-defined in both PDC channels) and refraction inside the crystal.

The interaction Hamiltonian of parametric down-conversion is given by[17]

$$\hat{V}(t) = \alpha \int \tilde{K}(\mathbf{r}) \hat{\tilde{E}}_t^{(-)}(\mathbf{r},t) \hat{\tilde{E}}_s^{(-)}(\mathbf{r},t) \tilde{E}_p^{(+)}(\mathbf{r},t) d^3r \ + \text{H.c.}, \tag{1}$$

where α is proportional to the second order nonlinear susceptibility and is assumed frequency independent, $\tilde{K}(\mathbf{r})$ describes the nonlinear crystal volume and is one inside and zero outside the crystal. We treat the fields in the signal (s) and trigger (t) channels as quantum operators, with their positive-frequency components given by

$$\hat{\tilde{E}}_{s,t}^{(+)}(\mathbf{r},t) = \int e^{-i(\mathbf{k}_{s,t}\cdot\mathbf{r} - \omega_{s,t} t)} \hat{a}_{\mathbf{k}_{s,t},\omega_{s,t}} d^3k_{s,t}\, d\omega_{s,t}; \tag{2}$$

the coherent pump field is treated classically:

$$\tilde{E}_p^{(+)}(\mathbf{r},t) = \int E_p^{(+)}(\mathbf{k}_p, \omega_p) e^{i(\mathbf{k}_p\cdot\mathbf{r} - \omega_p t)} d^3k_p\, d\omega_p\,. \tag{3}$$

For all fields, quantum or classical, the Hermitian electric field observable is written as $\tilde{E}_p(\mathbf{r},t) = \tilde{E}_p^{(+)}(\mathbf{r},t) + \tilde{E}_p^{(-)}(\mathbf{r},t)$, with $\tilde{E}_p^{(-)}(\mathbf{r},t) = (\tilde{E}_p^{(+)}(\mathbf{r},t))^\dagger$.

Assuming the signal and trigger modes to be initially in the vacuum state and restricting the consideration to the first order perturbation theory, we write the resulting biphoton state as

$$|B\rangle = |0\rangle_s |0\rangle_t - i \int_{-\infty}^{\infty} \hat{V}(t)\, dt. \tag{4}$$

*Here an atomic ensemble was assumed to play the role of the nonlinear medium.

Performing the integration we obtain:

$$|B\rangle = |0\rangle_s|0\rangle_t - i\int d^3k_s\, d\omega_s\, d^3k_t\, d\omega_t\, \Psi(\mathbf{k}_s,\omega_s,\mathbf{k}_t,\omega_t)|1_{\mathbf{k}_s,\omega_s}\rangle_s|1_{\mathbf{k}_t,\omega_t}\rangle_t, \tag{5}$$

with

$$\Psi(\mathbf{k}_s,\omega_s,\mathbf{k}_t,\omega_t) = \alpha\int E_p^{(+)}(\mathbf{k}_p,\omega_s+\omega_t)K(\Delta\mathbf{k})d^3k_p. \tag{6}$$

Here $K(\mathbf{k})$ is the Fourier transform of $\tilde{K}(\mathbf{r})$ and the $\mathbf{k}$-vector mismatch is $\Delta\mathbf{k} = \mathbf{k}_p - \mathbf{k}_s - \mathbf{k}_t$.

The trigger photon is then selected by spatial and frequency filters and is detected by a single-photon counter. Conditioned on the detection event the non-local biphoton state collapses into a single photon state in the signal mode. The properties of this mode are determined by the optical mode of the pump photon and the spatial and spectral filtering in the trigger channel:

$$\hat{\rho}_s = \mathrm{Tr_t}(\hat{\rho}_\mathrm{t}|\mathrm{B}\rangle\langle\mathrm{B}|), \tag{7}$$

where the trace is taken over the trigger states and $\hat{\rho}_t$ denotes the state ensemble selected by the filters:

$$\hat{\rho}_t = \int T(\mathbf{k}_t,\omega_t)|1_{\mathbf{k}_t,\omega_t}\rangle_t\langle 1_{\mathbf{k}_t,\omega_t}|_t\, d^3k_t\, d\omega_t \tag{8}$$

with $T(\mathbf{k},\omega)$ being the spatiotemporal transmission function of the filters.

An explicit calculation of the quantum state (7) of the photon in the signal channel yields

$$\hat{\rho}_s = \int d^3k_s d\omega_s d^3k'_s d\omega'_s \Phi(\mathbf{k}_s,\omega_s,\mathbf{k}'_s,\omega'_s)\big|1_{\mathbf{k}'_s,\omega'_s}\big\rangle_s\langle 1_{\mathbf{k}_s,\omega_s}|_s, \tag{9}$$

where

$$\Phi(\mathbf{k}_s,\omega_s,\mathbf{k}'_s,\omega'_s) = |\alpha|^2\int d^3k_t\, d\omega_t\, d^3k_p\; d^3k'_p\; E_p^{(-)}(\mathbf{k}_p,\omega_s+\omega_t)E_p^{(+)}(\mathbf{k}'_p,\omega'_s+\omega_t)\, T(\mathbf{k}_t,\omega_t)K^*(\Delta\mathbf{k})K(\Delta\mathbf{k}'), \tag{10}$$

with $\Delta\mathbf{k}$ as above and $\Delta\mathbf{k}' = \mathbf{k}'_p - \mathbf{k}'_s - \mathbf{k}_t$.

3. MODELING THE SINGLE PHOTON MODE WITH A CLASSICAL WAVE

We now calculate the field correlation function $\Gamma(\mathbf{k},\omega,\mathbf{k}',\omega') = < E^{(-)}(\mathbf{k},\omega)E^{(+)}(\mathbf{k}',\omega') >$ of the difference frequency (DFG) wave generated through nonlinear interaction of the advanced wave and the pump. For the nonlinear polarization inside the crystal, we write

$$\tilde{P}_{\mathrm{DFG}}(\mathbf{r},t) \propto \tilde{E}_A(\mathbf{r},t)\tilde{E}_p(\mathbf{r},t). \tag{11}$$

Here $\tilde{E}_p(\mathbf{r},t)$ and $\tilde{E}_A(\mathbf{r},t)$ are the electric fields of the pump and advanced waves, respectively. The mode of the DFG field is obtained from Eq. (11) via a Fourier transform which is restricted to the crystal volume:

$$E_{\mathrm{DFG}}^{(+)}(\mathbf{k}_s,\omega_s) = \beta\,\delta(k_s-\omega_s/c)\int d^3k_A\, d\omega_A\, d^3k_p E_A^{(-)}(\mathbf{k}_A,\omega_A)\, E_p^{(+)}(\mathbf{k}_p,\omega_s+\omega_A)K(\Delta\mathbf{k}). \tag{12}$$

The proportionality coefficient β represents the nonlinearity of the medium, $\Delta\mathbf{k} = \mathbf{k}_p - \mathbf{k}_s - \mathbf{k}_A$. If the advanced wave field is partially incoherent and is characterized by a correlation function $\Gamma_A(\mathbf{k}_A,\omega_A,\mathbf{k}'_A,\omega'_A)$, the above equation generalizes to

$$\begin{aligned}\Gamma_{\mathrm{DFG}}(\mathbf{k}_s,\omega_s,\mathbf{k}'_s,\omega'_s) \;&=\; |\beta|^2\,\delta(k_s-\omega_s/c)\,\delta(k'_s-\omega'_s/c)\int d^3k_A\, d\omega_A\, d^3k'_A\, d\omega'_A\, d^3k_p\, d^3k'_p \\ &\times E_p^{(-)}(\mathbf{k}_p,\omega_s+\omega_A)E_p^{(+)}(\mathbf{k}'_p,\omega'_s+\omega'_A)\Gamma_A^*(\mathbf{k}_A,\omega_A,\mathbf{k}'_A,\omega'_A)K^*(\Delta\mathbf{k})K(\Delta\mathbf{k}').\end{aligned} \tag{13}$$

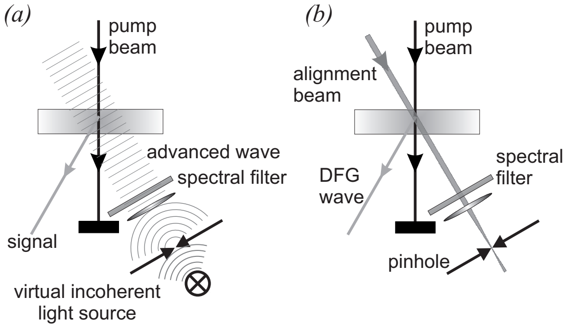

Figure 2. (a) Interaction of Klyshko's advanced wave with the pump generates a DFG mode that mimics that of the CPP. (b) In an experiment, the role of the advanced wave can be played by a laser beam, aligned so as to obtain maximum transmission through all the filters.

We define the correlation function of the advanced wave is in the following fashion. Suppose the single photon detector is replaced by an incoherent source continuously emitting omnidirectional incoherent light into a wide spectral range backwards in time. This completely incoherent light is characterized by the correlation function $\Gamma_0(\mathbf{k}', \omega', \mathbf{k}, \omega) \propto \delta^{(3)}(\mathbf{k}' - \mathbf{k})\, \delta(\omega' - \omega)$ which, upon passing through the spatial and spectral filters, transforms into

$$\Gamma_A(\mathbf{k}', \omega', \mathbf{k}, \omega) = T(\mathbf{k}, \omega)\, \delta^{(3)}(\mathbf{k}' - \mathbf{k})\, \delta(\omega' - \omega). \tag{14}$$

The advanced wave then enters the nonlinear crystal and interacts with the pump wave whenever and wherever it is present in the crystal. The nonlinear interaction of Klyshko's advanced wave with the pump pulse produces a pulse of DFG emission into the signal channel (Fig. 2(a)). Substituting the correlation function (14) of the advanced wave into Eq. (13) as Γ_A we find that

$$\Gamma_{\mathrm{DFG}}(\mathbf{k}_s, \omega_s, \mathbf{k}'_s, \omega'_s) \equiv \Phi(\mathbf{k}_s, \omega_s, \mathbf{k}'_s, \omega'_s). \tag{15}$$

The correlation function of the DFG pulse generated through the nonlinear interaction of the advanced wave and the pump pulse is identical to the density matrix of the single photon prepared by conditional measurements on a biphoton performed in the same optical arrangement[†].

4. DISCUSSION

Unlike Klyshko, who said that the advanced wave is a δ-function pulse, we consider it to be a continuous, partially incoherent wave. The duration of the advanced wave is in fact determined by the uncertainty of the photon arrival time measurement. With modern detectors, it amounts to at least tens of picoseconds. If the down-conversion experiment is performed in an ultrashort pulsed setting, this uncertainty substantially exceeds the pump pulse width, so the advanced wave can be considered continuous. On the other hand, if the pump laser is continuous, the situation is more complicated and the timing uncertainty must be taken into account more rigorously in order to determine the correct correlation function of the DFG wave and the density matrix of the conditional single photon.

Can a completely incoherent advanced wave give rise to a highly coherent, transform-limited difference frequency pulse? The answer is positive provided that the filtering in the trigger channel is sufficiently narrow.[5] Indeed, when propagating through the spectral filter, the initially incoherent advanced wave acquires some finite degree of temporal coherence, quantified by the coherence time equal to the inverse

[†]The delta-functions, included into Eq. (13) to eliminate nonphysical Fourier components of the DFG field, are also implicitly present in Eqs. (9) and (10) as the single-photon states $|1_{\mathbf{k}_s,\omega_s}\rangle_s$ exist only when $k_s = \omega_s/c$.

filter width $\tau_c = \sigma_t^{-1}$. Further, we notice that the nonlinear interaction between the advanced wave and the pump is restricted by the time window determined by the duration τ_p of the pump pulse. If the latter is much shorter than τ_c, the advanced wave can be considered almost coherent within this window, and the DFG pulse is almost transform limited. Reformulating this in the language of single photons, we find that in order to obtain a conditional photon in a pure temporal mode, the filter in the trigger channel must be much narrower than the inverse pulse width: $\sigma_t \ll \tau_p^{-1}$. This result confirms the conclusion of Ou.[18]

Same considerations are valid for spatial coherence. As the advanced wave passes through a narrow aperture, it gains some degree of transverse coherence according to the Van Cittert-Zernike theorem. Because the nonlinear interaction is restricted to the area where the pump field is present, the resulting signal (DFG) field is also partially coherent provided the pump beam diameter is smaller than the coherence width of the advanced wave in the plane of the crystal.[5]

The identity (15) can be easily generalized to optical filters of random configuration, more complex than a combination of spatial and spectral filters described by Eqs. (8) and (14). Its applicability is also independent from other features of the experimental setup, such as the configuration of PDC, properties of the pump beam, geometry of the crystal, walk-off and group velocity dispersion effects, *etc.* and appears to be very general. The only restriction that has to be taken into account is the first order perturbation theory, which implies that the probability of generating two or more biphotons at a time is negligible.

By varying the configuration of the filter in the trigger channel one has some freedom in forming the CPP mode with the required spatiotemporal properties. This possibility can be considered as an example of *remote state preparation* in the sense discussed by Bennett *et al.*[19] The original biphoton state is highly entangled in the frequency-momentum space and this entanglement plays an essential role in generating the Fock state. The signal mode *does not exist* unless and until the trigger photon passes through the filters and is registered. A detection event results in a non-local preparation of a single photon in an optical mode whose characteristics are determined by the way in which the measurement in the trigger channel is performed.

Apart from its theoretical implications, the result (15) finds its use in experimental practice, namely when a need arises to model a CPP with a classical wave. This is necessary, for example, when the mode of the photon needs to be matched to a classical mode,[3] or to that of another CPP from a different source.[13] The traditional procedure of matching two classical modes with each other — by observing interference fringes and optimizing their visibility — is not applicable to the situation when one of the modes is a single photon. There is no laser beam to mode match to. The only information available to the experimentalist is the remote location and the width of the trigger filter and the parameters of the pump. Although the spatial location of the CPP can be approximately determined by detecting coincidences between the photon count events in the signal and trigger,[20] optimizing the mode matching requires adjustment of a much larger set of degrees of freedom, such as the beam direction, divergence, spatial and temporal width, optical delay, etc. Reliable adjustment of these parameters cannot be achieved through sole optimization of the coincidence rate.

This is where the Klyshko model comes into play. Although the advanced wave prolagates backwards in space and time and is thus a purely imaginary object, it can be modeled by an *alignment beam* inserted into the trigger channel so that it overlaps spatially and temporally with the pump beam inside the crystal and passes through the optical filters (Fig. 2 (b)). Nonlinear interaction of such an alignment beam with the pump wave will produce difference frequency generation into a spatiotemporal mode similar (albeit no longer completely identical) to that of the CPP.

As mentioned in the Introduction, the universality of the result (15) permits its application in the analysis of many experimental settings involving parametric down-conversion. One obvious application area is quantum coincidence imaging.[15, 16] Employing the advanced wave model allows one to analyze the quantum image formation in a completely classical view frame of geometrical optics, and helps one solve frequently debated issues such as the role of entanglement[21] or the configuration of the photon detector in the trigger channel, as well as evaluate the resolution and coherence properties of the image.

A perhaps less straightforward application of the advanced wave model is multimode entanglement. In the case of polarization entanglement, for example, detection of a photon of a particular polarization in the trigger channel remotely prepared a photon of the same polarization in the signal channel, leading to a violation of the Bell inequality.[9] In the framework of the advanced wave model, detection of a photon with a certain polarization is equivalent to emission of an advanced wave with this polarization. If the spatial mode of the advanced wave lies within one of the emission cone intersection areas,[9] both its polarization components will interact with the pump wave, giving rise to a DFG wave also containing two polarization components of the same relative intensity. In a similar manner, one can understand other multimode entanglement experiments, such as angular-momentum[11] or time-of-arrival[12] entanglement.

In summary, we have investigated the spatiotemporal optical mode of the single-photon Fock state prepared by conditional measurements on a biphoton and found it to be identical to that of a classical wave generated due to a nonlinear interaction of the pump wave and Klyshko's advanced wave. We discussed the applicability of this identity in various experimental settings.

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REFERENCES

1. C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986)
2. P. Grangier, G. Roger and A. Aspect, Europhys. Lett. **1**, 173 (1986)
3. A. I. Lvovsky *et al*, Phys Rev. Lett. **87**, 050402 (2001)
4. S. A. Babichev, B. Brezger, A. I. Lvovsky, Phys Rev. Lett. **92**, 047903 (2004); S. A. Babichev, J. Appel, A. I. Lvovsky, *ibid.* **92**, 193601 (2004); A. I. Lvovsky, J. Mlynek, *ibid.* **88**, 250401 (2002)
5. T. Aichele, A. I. Lvovsky and S. Schiller, Eur. Phys. J. **D 18**, 237 (2002).
6. D. N. Klyshko, Phys. Lett. **A 128**, 133 (1988);
7. D. N. Klyshko, Phys. Lett. **A 132**, 299 (1988);
8. D. N. Klyshko, Sov. Phys. Usp., **31** 74, (1988).
9. P. G. Kwiat *et al.*, Phys. Rev. Lett. **75**, 4337 (1995)
10. J. D. Franson Phys. Rev. Lett. **62**, 2205 (1989)
11. A. Mair *et al.*, Nature **412**, 313 (2001)]
12. J. Brendel *et al.*, Phys. Rev. Lett. **82**, 2594 (1999)
13. J.-W. Pan *et al.*, Phys. Rev. Lett. **80**, 3891 (1998)
14. J. C. Howell *et al.*, Phys. Rev. Lett. **92**, 210403 (2004)
15. T. B. Pittman *et al.*, Phys. Rev. A. **52**, 3429 (1995)
16. R. S. Bennik, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. **89**, 113601 (2002); R. S. Bennik, S. J. Bentley, R. W. Boyd, and J. C. Howell, *ibid.* **92**, 033601 (2004);
17. Z. Y. Ou, L. J. Wang, and L. Mandel, Phys. Rev. **A 40**, 1428 (1989)
18. Z. Y. Ou, Qu. Semiclass. Opt. **9**, 599 (1997)
19. C. H. Bennett *et al.*, Phys. Rev. Lett. **87**, 077902 (2001)
20. T. B. Pittman *et al.*, Phys. Rev. A 53, 2804-2815 (1996);
21. A. F. Abouraddy *et al.*, Phys. Rev. Lett. **87**, 123602 (2001)